Ballistic Pendulum Lab

The Conservation of Angular Momentum is probably the least familiar of the three great conservation laws for most people. In this lab a metal ball is fired horizontally into a swinging arm or "Ballistic Pendulum" and captured by a spring-loaded clasp near its lower end. The ball brings in energy of course. However, the collision is inelastic and an unknown portion of that energy is lost as heat. The ball also brings in linear momentum. Once again an unknown enters the problem in that the axle at the top end exerts a force on the pendulum and injects an unknown amount of momentum into the system. The trick here is that we can track the third conserved quantity – "angular momentum" – because, although the axle does exert a force – that force has no lever arm and thus no torque. The fundamental equation of motion:

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

tells us that torque "is" the rate of transfer of angular momentum, and that without torque the **<u>angular</u>** momentum must remain constant. This is what we use.

The Lab

This lab is conducted with the goal of finding the velocity of the incoming projectile (the steel ball). You will derive a (pretty involved) relationship from our conservation laws which tells you the incoming speed of the projectile from the maximum angle that the recoiling pendulum finally swings up to. When you measure this angle you stick it in this universal relationship and then … *voilà* !– you get the velocity (even for a high speed projectile like a bullet, which is how this was really used for a long time before the advent of highspeed electronics [think U.S. Civil War etc.]).
Our attention focuses on three individual "*moments-in-time*" in the full process:

Moment #1) *Infinitesimally* "before the collision". The incoming <u>angular momentum</u> is given by:

$$L_{in} = mvr$$

Moment #2) *Infinitesimally* "after the collision". The outgoing <u>angular momentum</u> is given by:

$$L_{out} = I\omega_{out}$$

Moment #3) At the moment of maximum angular swing. At this final moment we have:

$$KE_{out} = PE_{final}$$

So here's the thought process. At moment #1 the ball (a particle) is bringing in angular momentum and we have expressed it in the form appropriate for a particle. Directly after the collision (about a thousandth of a second later) … the projectile has "joined" the pendulum in an inelastic collision. Nothing has moved almost at all – but a lot has changed!

At Moment #2 the projectile-pendulum "System" is now swinging together "as one" with an angular velocity ω_{out} and this "System" has **exactly** the Angular-Momentum that the projectile initially was carrying. Nothing else but *Angular Momentum* is "continuous across the duration" of the collision. But now from Moment #2 until Moment #3 … we have no more collisions and we can follow the Mechanical Energy which is conserved as gravity exerts itself on the system: *"doing work, transferring momentum and exerting torque"*. At Moment #2 all energy is Kinetic Energy which at Moment #3 has all, finally, become gravitational Potential Energy. The basic conservation of energy statement may be written as

$$KE_{out} = PE_{final}$$

which may be further written as

$$\frac{{L_{out}}^2}{2I} = Mg \cdot \Delta h_{CM}$$

And since $L_{out} = L_{in}$ by the assumption of conservation of angular momentum, we may now write

$$\frac{{L_{in}}^2}{2I} = Mg \cdot \Delta h_{CM}$$

And finally, this becomes:

$$\frac{(mvr)^2}{2I} = Mg \cdot \Delta h_{CM}$$

Convince yourself that the Center of Mass has risen by $\Delta h_{CM} = R_{CM}(1 - \cos(\theta_{max}))$.

The moment of Inertia "I" can be found from a second simple measurement … that of the period of swing " T " of the "System" now used as a *simple pendulum*. This a result we will deduce in our next unit of study.

The necessary relationship given here is:

$$T = 2\pi\sqrt{\frac{I}{MgR_{cm}}}$$

We will need the numbers: { m, M, g, T, r, R_{CM}, θ_{max}} . From {T,M, g, R_{CM}} we can get I.
Then from θ_{max} and R_{CM} we can get h_{CM} and from this we can work our way back to the incoming velocity v .

Your task in this lab is then to determine the following :

1) Produce a single algebraic expression giving v as a function of the other quantities. A really cute trick happens if you use the trig identity: $1 - \cos(\theta) = 2sin^2(\theta/2)$
2) Insert the measured data and produce the required velocity v.
3) Check this velocity against the velocity deduced from the range of the same projectile fired off the table top (just as we did in the beginning of the semester) and find the % difference. Are we within expected uncertainty ? (i.e. did it work ?)

4) Deduce the percentage of the Kinetic Energy lost in the inelastic collision.

5) Deduce the percentage of the Linear Momentum lost in the inelastic collision. This momentum transfer occurred at the axle. You need to recall that the incoming linear momentum is simply given by $P_{in} = mv$, while the outgoing linear momentum at Moment #2 is given by

$$P_{out} = MV_{CM} \;= M\omega_{out}R_{CM}$$

If axles aren't strong enough and can't deliver the momentum required of them … they rip free in such circumstances! That's bad engineering! If the incoming projectile was moving left to right … did the axle push on the projectile to the left or to the right?

The Data:

Next, we present actual data taken in that lab period.

Projectile Mass m = .0667 kg

"System" Mass M = .3092 kg This is the swinging arm plus the projectile.

Lever arm r = .30 meter

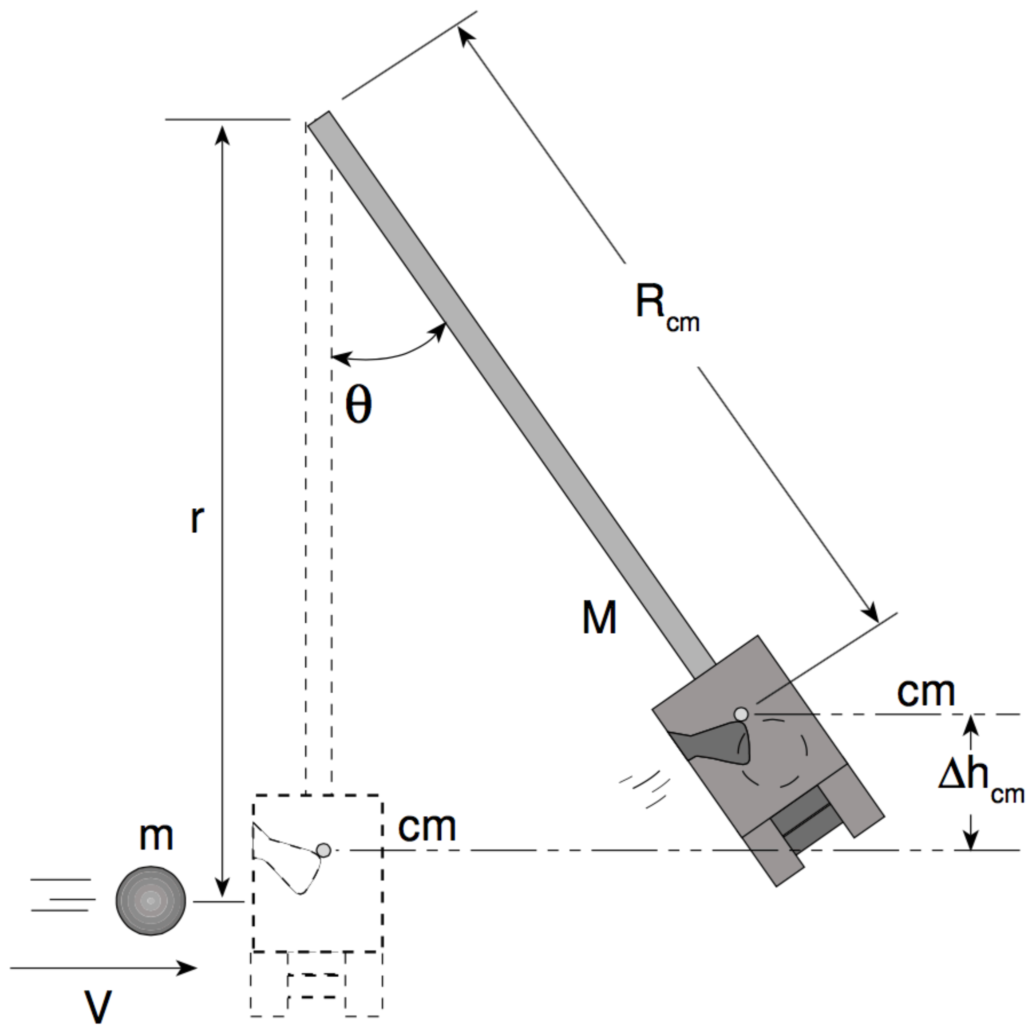
Center of Mass R_{cm} = .285 meter (this is the distance from the axle out to the CM)

Period of Swing T = 1.126 second

Max. Angle θ_{max} = 29^{o}

Height of cannon off the floor for the test shots: h = 0.895 meter

Test Trial off table top	**Range** (meters)
1	1.567
2	1.570
3	1.604
4	1.610



R
cm
θ
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M
cm
Δh
cm
m
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V